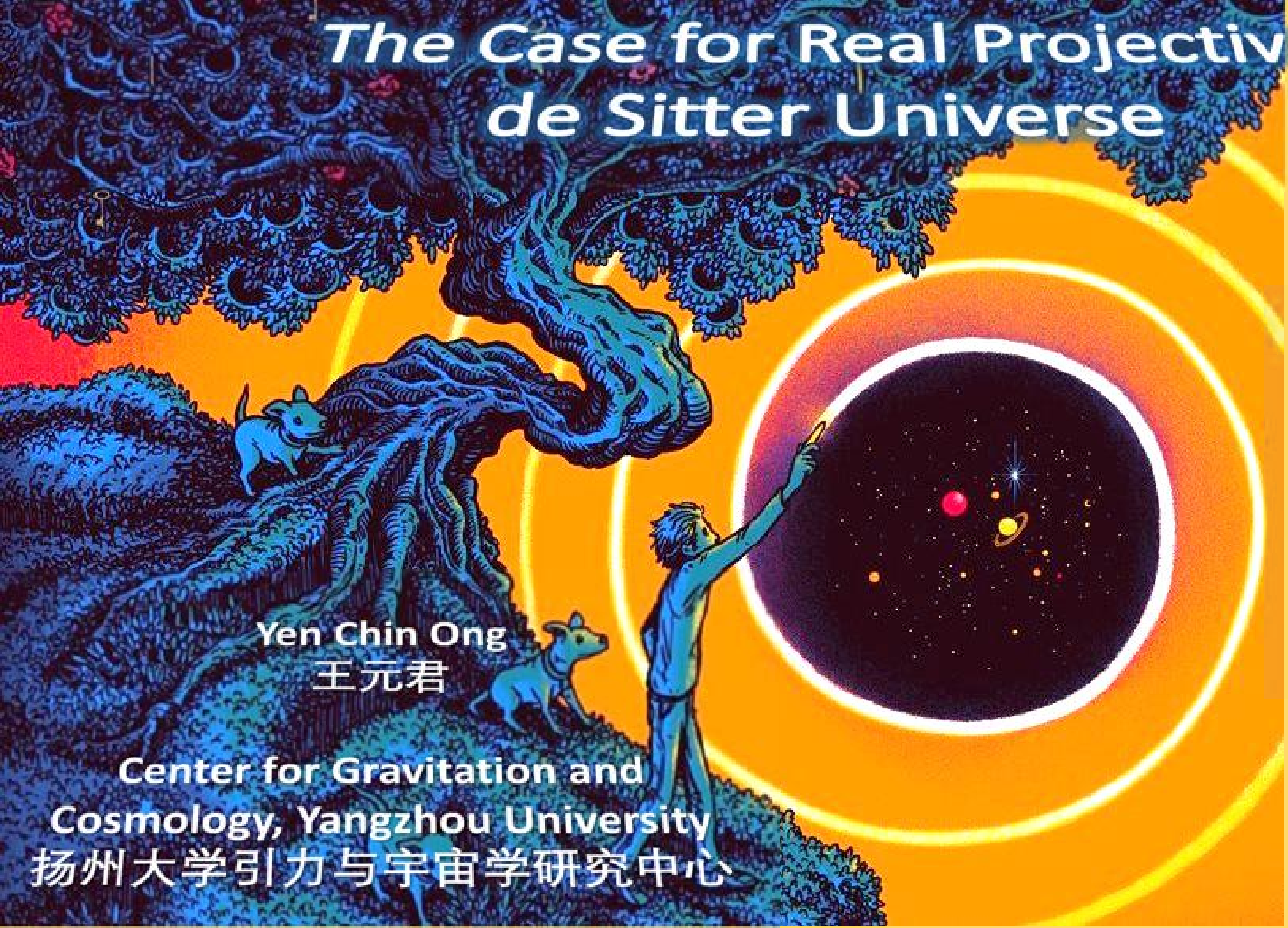
The Case for Real Projective de Sitter Universe



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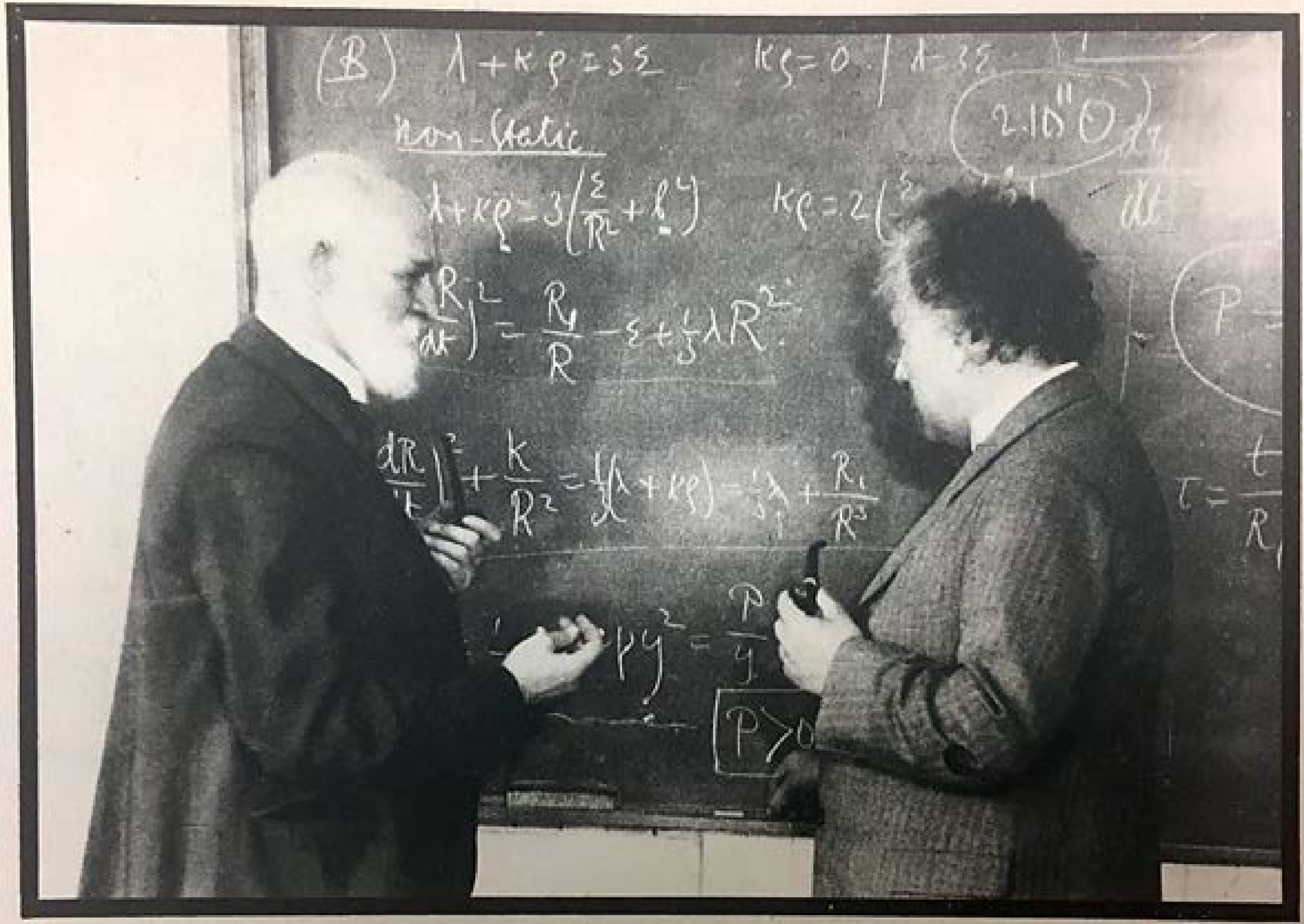


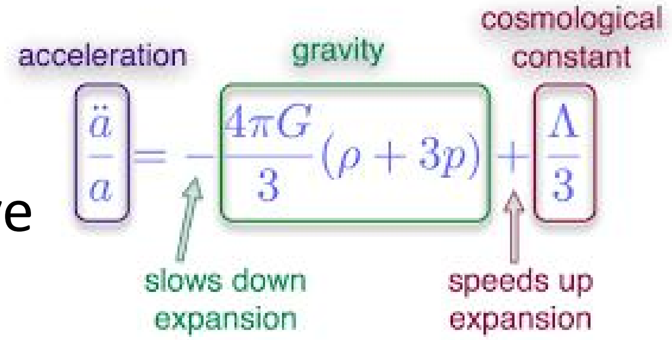
Keystone

WILLEM DE SITTER AND ALBERT EINSTEIN

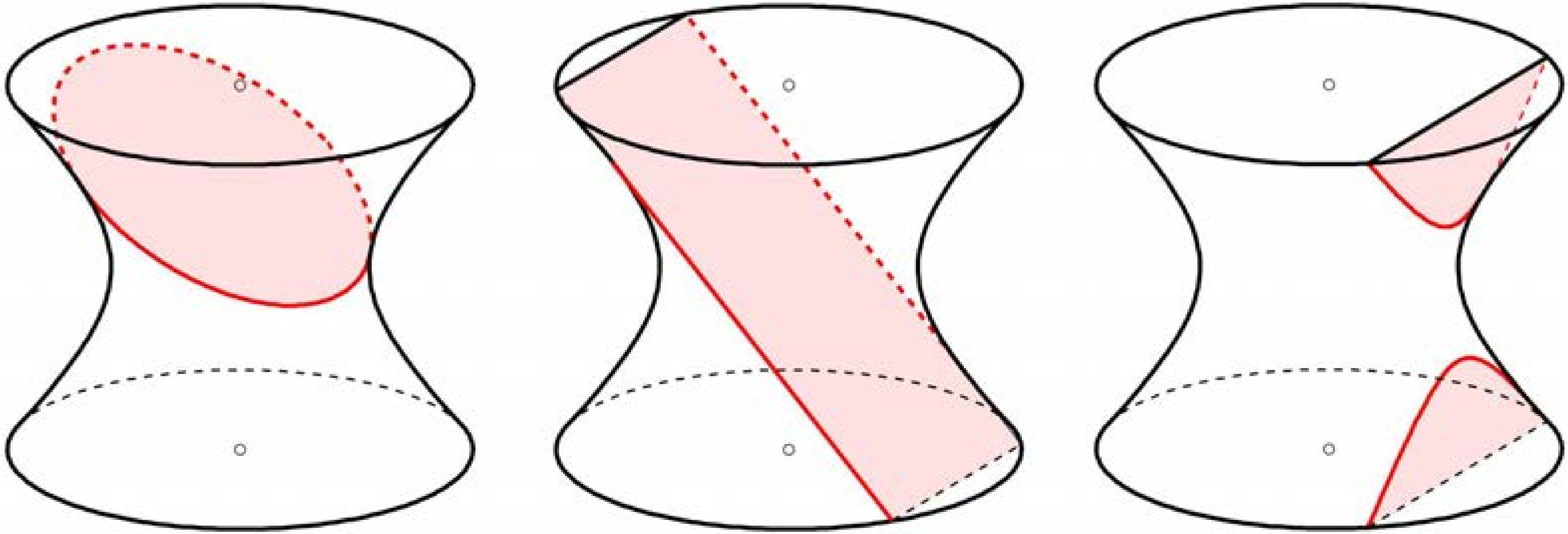
. . . revising their theories on a Cal Tech blackboard, to accord with Dr. Hubble's discoveries in the sky.

What is de Sitter Space?

Maximally symmetric solution to Einstein Field Equations with positive cosmological constant.



Global topology: $\mathbb{R} \times S^3$

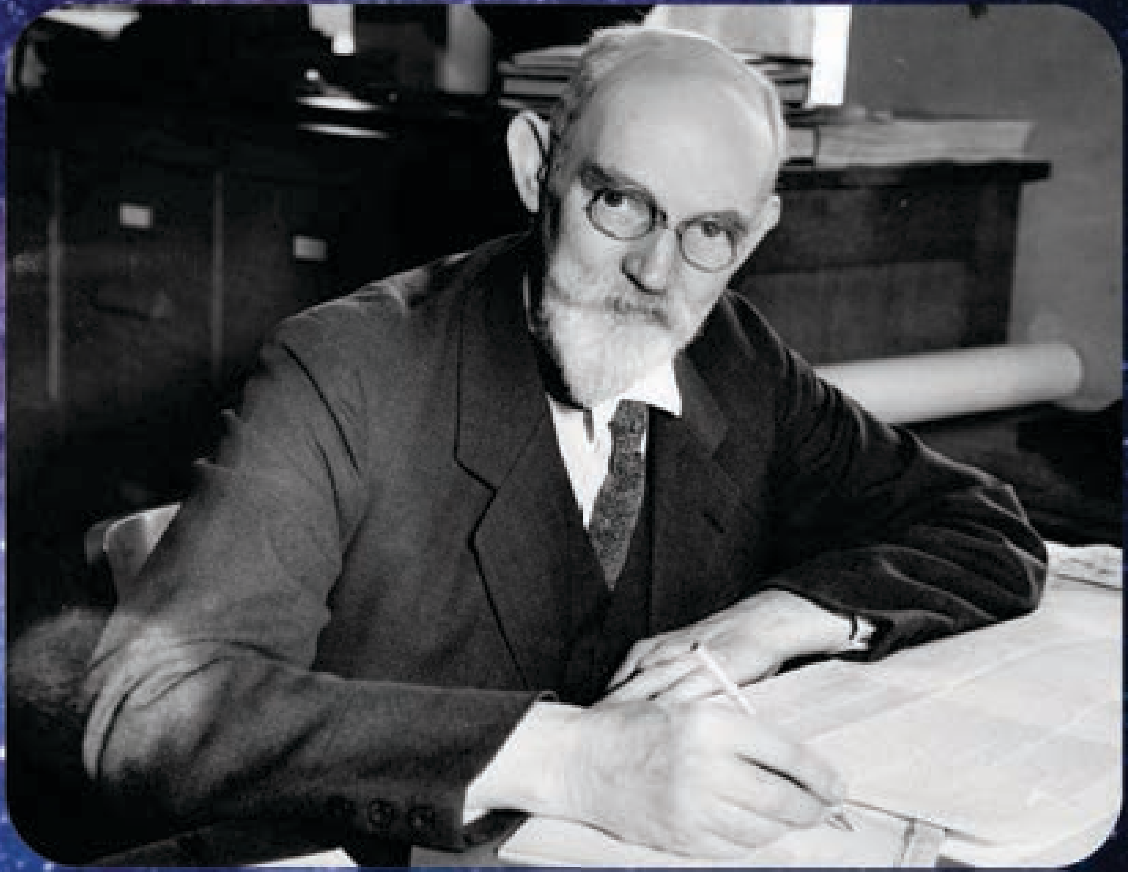


Closed spherical universe
Positive 3d curvature

Open flat universe
Zero 3d curvature

Open hyperboloid universe
Negative 3d curvature

Schwarzschild-de Sitter according to Schwarzschild and de Sitter



W. de Sitter, *Einstein's theory of gravitation and its astronomical consequences, Third Paper,* *Mon. Not. Roy. Astron. Soc.* **78** (1917) 3 [INSPIRE].

K. Schwarzschild, *Über das zulässige Krümmungsmass des Raumes, Vierteljahrschrift d. Astronom. Gesellschaft* **35** (1900) 337.

K. Schwarzschild, *On the Permissible Curvature of Space, Class. Quant. Grav.* **15** (1998) 2539.

Usually cited as one of the first attempts to discuss, from an observational point of view, the possibility that the spatial sections of the Universe may not have the geometry or topology of ordinary Euclidean space.

When Schwarzschild discussed spaces with positive curvature, he *only* discussed the real projective space $\mathbb{RP}^3$: "the simplest of the spaces with spherical trigonometry."

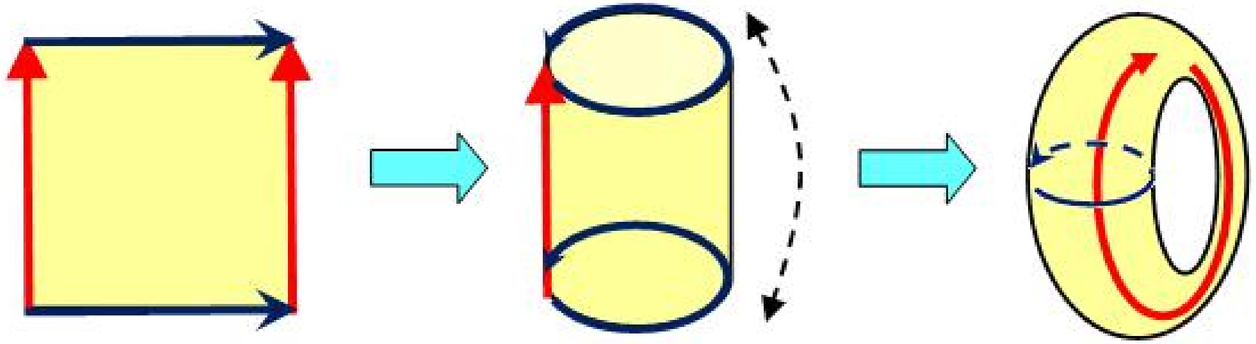
de Sitter, citing Schwarzschild: "[...] is really the simpler case, and it is preferable to adopt this for the physical world."

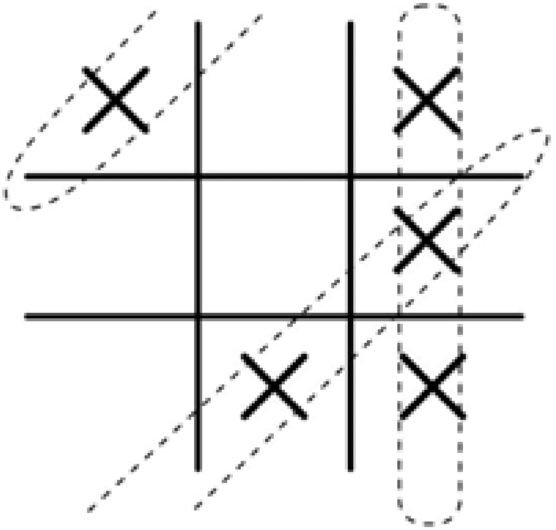
What is Real Projective Space, and why is it “simpler”?

Start with 2 dimensions:

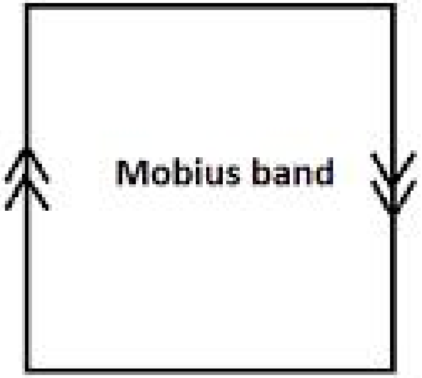
Torus:

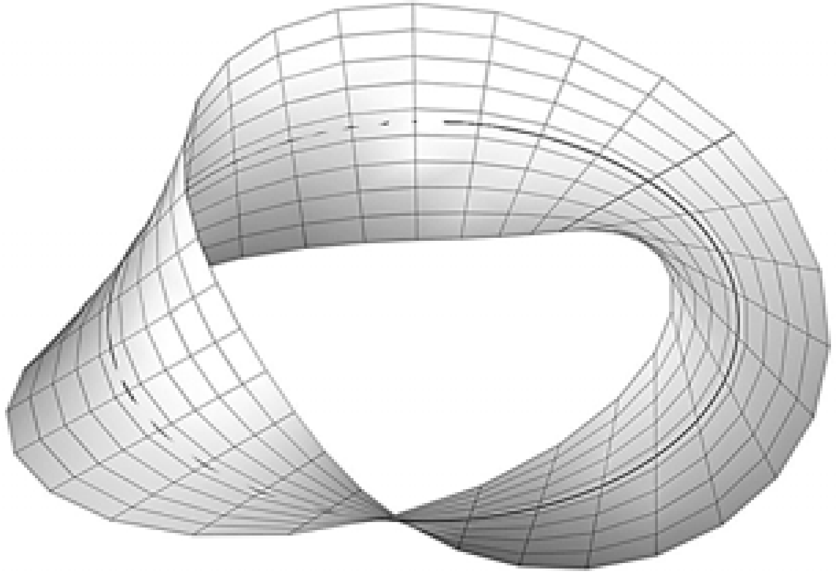
$T^2 \cong S^1 \times S^1$



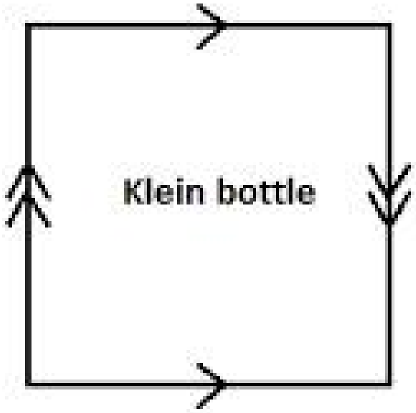


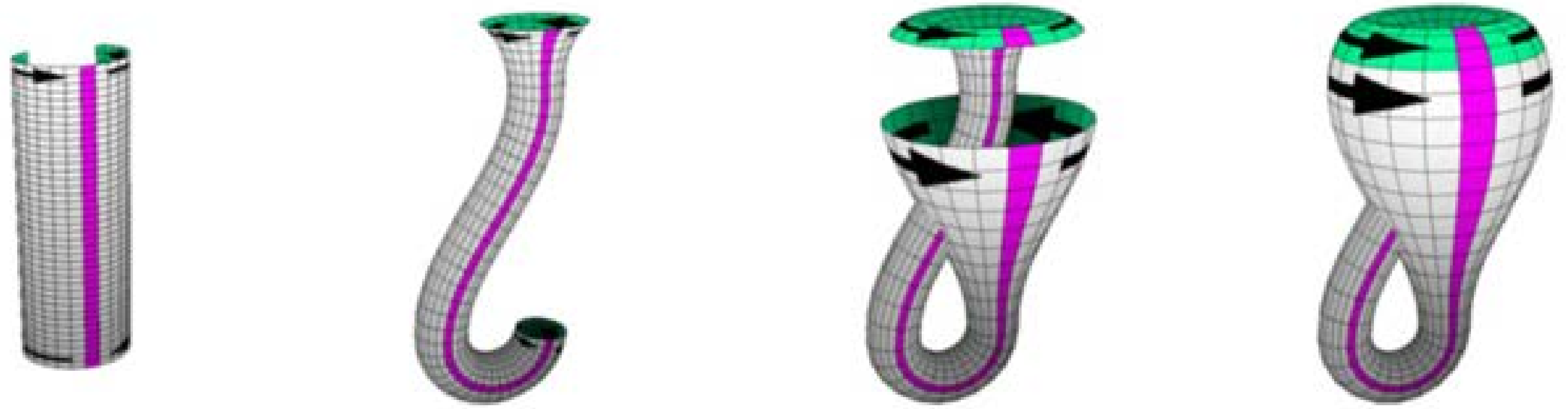
Möbius strip:

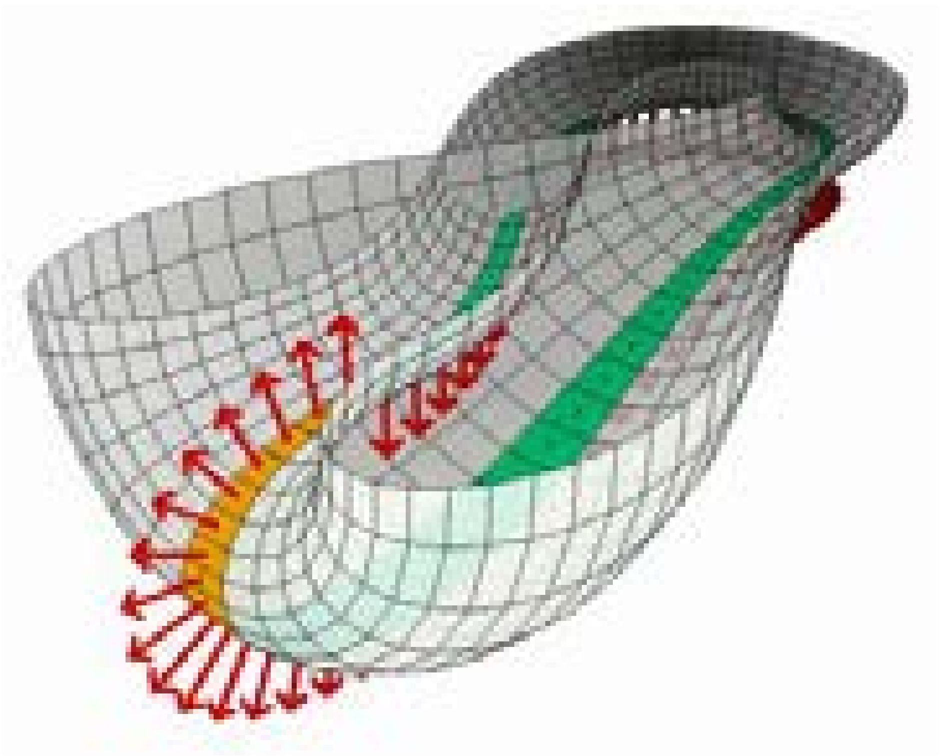




Klein Bottle:

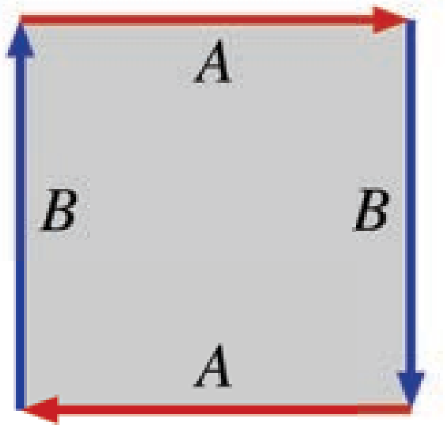




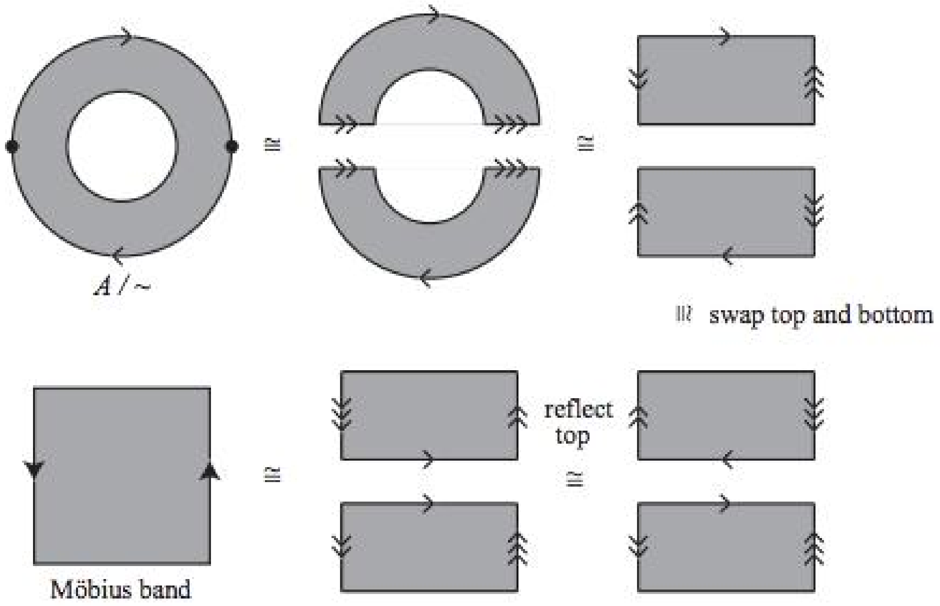


What is Real Projective Space, and why is it “simpler”?

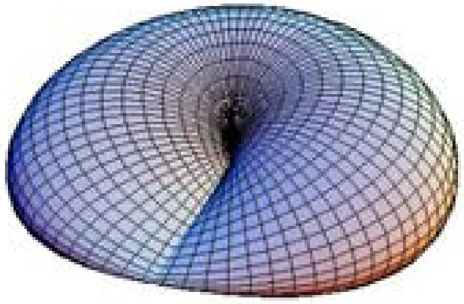
Real Projective Space:

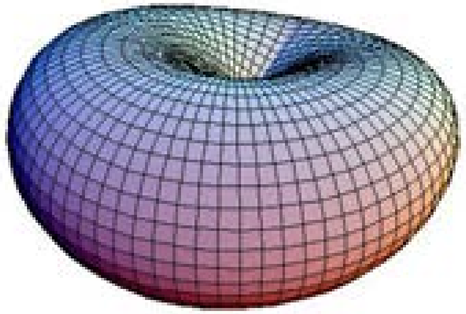


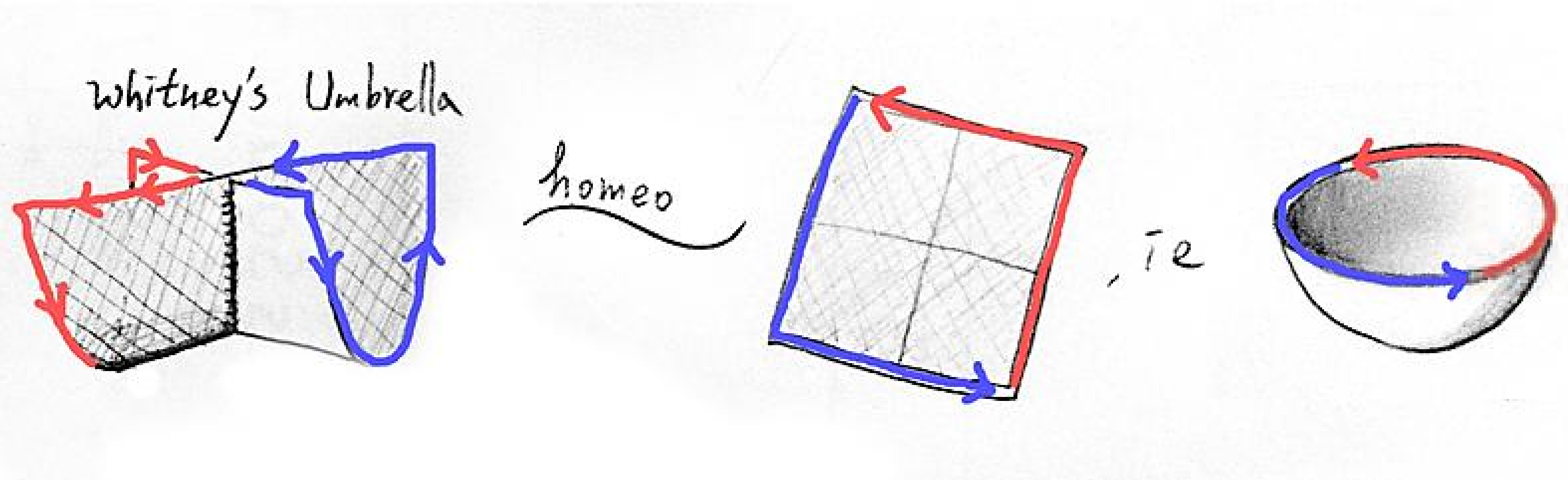
Real Projective Space minus a disk is just Möbius strip:



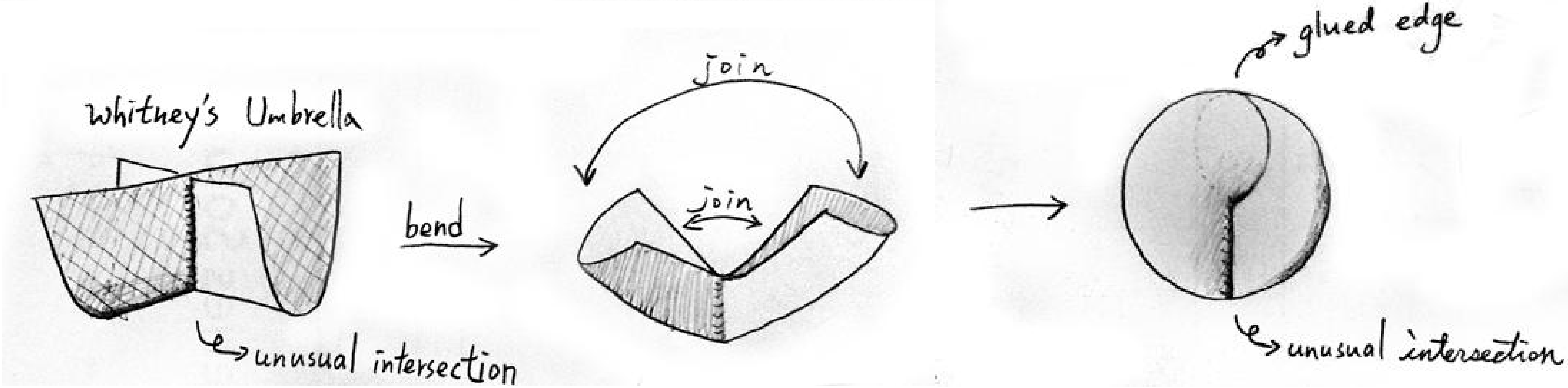
A Visualization: Cross Cap





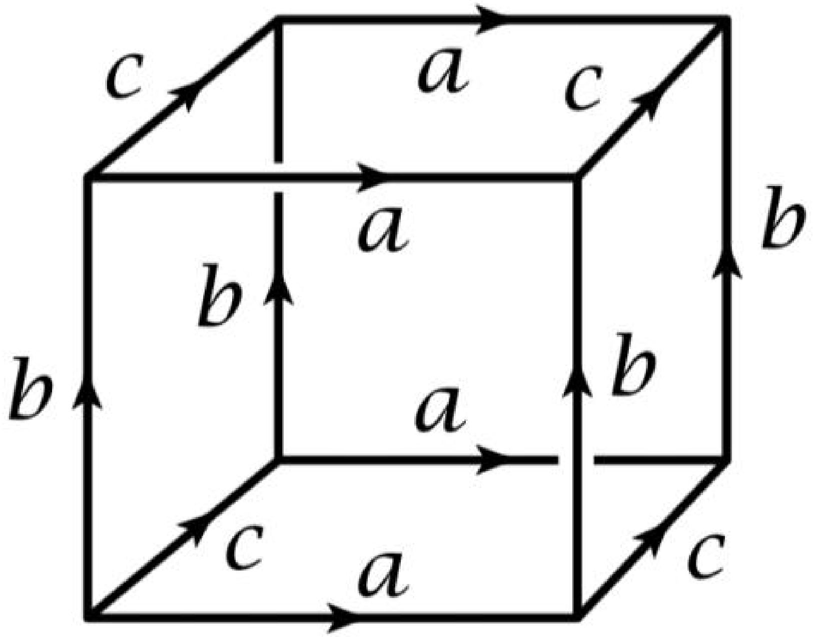


Note: $\mathbb{R}P^2 \cong S^2/\mathbb{Z}_2$



3-dimensions

$T^3 \cong S^1 \times S^1 \times S^1$



$\mathbb{RP}^3 \cong S^3/\mathbb{Z}_2$: note that unlike $\mathbb{RP}^2$, $\mathbb{RP}^3$ is orientable.

Indeed $\mathbb{RP}^3$ is quite special: $\mathbb{RP}^{2n}, n \geq 1$ are non-orientable; $\mathbb{RP}^{1+4n}, n \geq 1$ are orientable but not a spin manifold.

How is this SIMPLER ?!

Schwarzschild *explicitly* rejects S^3 -- "one would not consider such complicated (sic) assumptions unless it were really necessary" because light emitted from a point in S^3 would collect again at the antipode.

$\mathbb{RP}^3$ is more complicated mathematically but simpler physically:

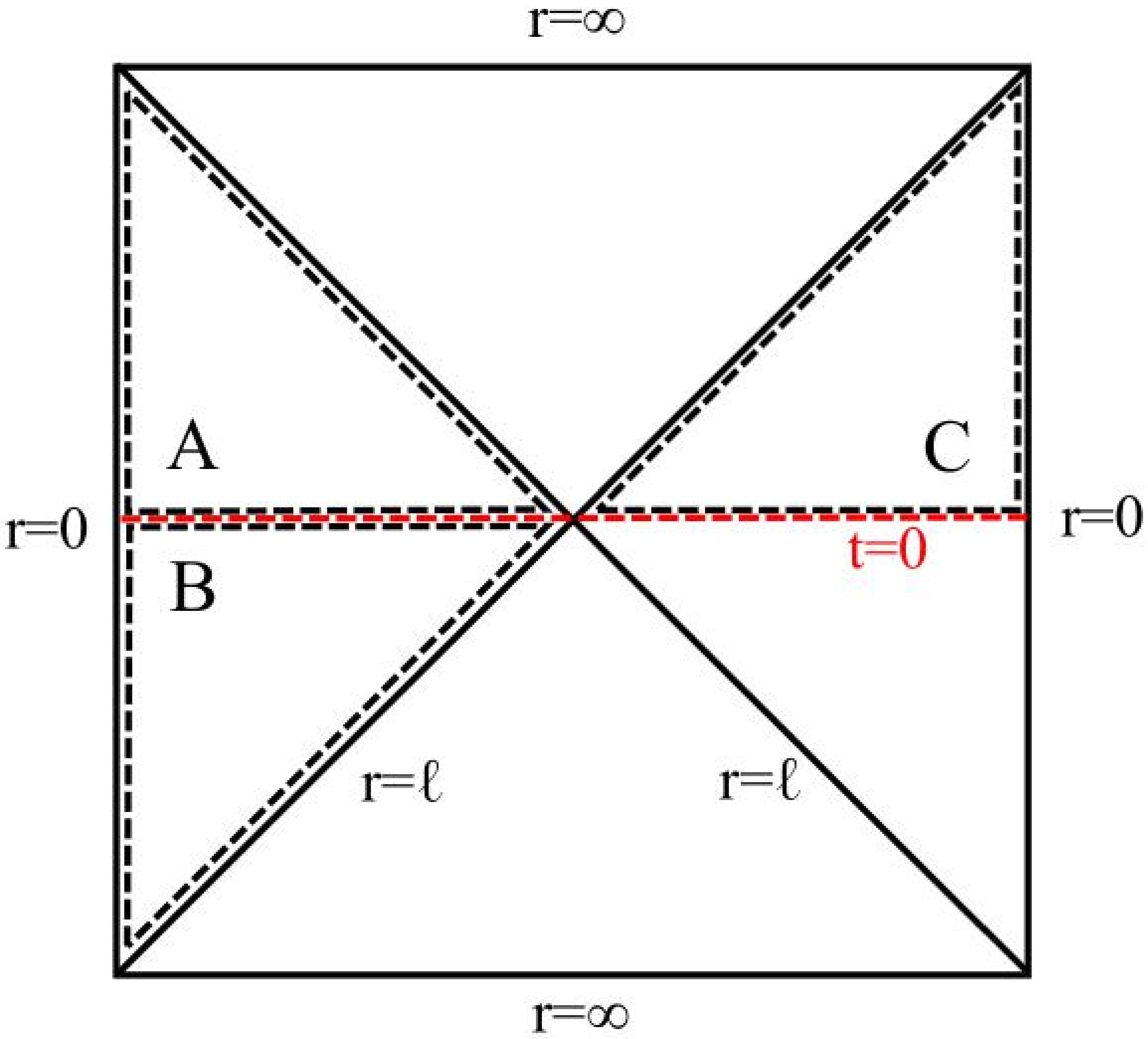
> Schwarzschild's point is that any two coplanar geodesics in $\mathbb{R}P^3$ intersect only once, while in S^3 they would do so twice. The intersection of geodesics is however a matter of local physics, and it is absurd that the geometry should try to enforce correlations on the largest possible length scales. (In modern language, Schwarzschild had concerns about locality of physics.)

B. McInnes, *de Sitter and Schwarzschild-de Sitter according to Schwarzschild and de Sitter*, *JHEP* **09** (2003) 009 [hep-th/0308022] [INSPIRE].

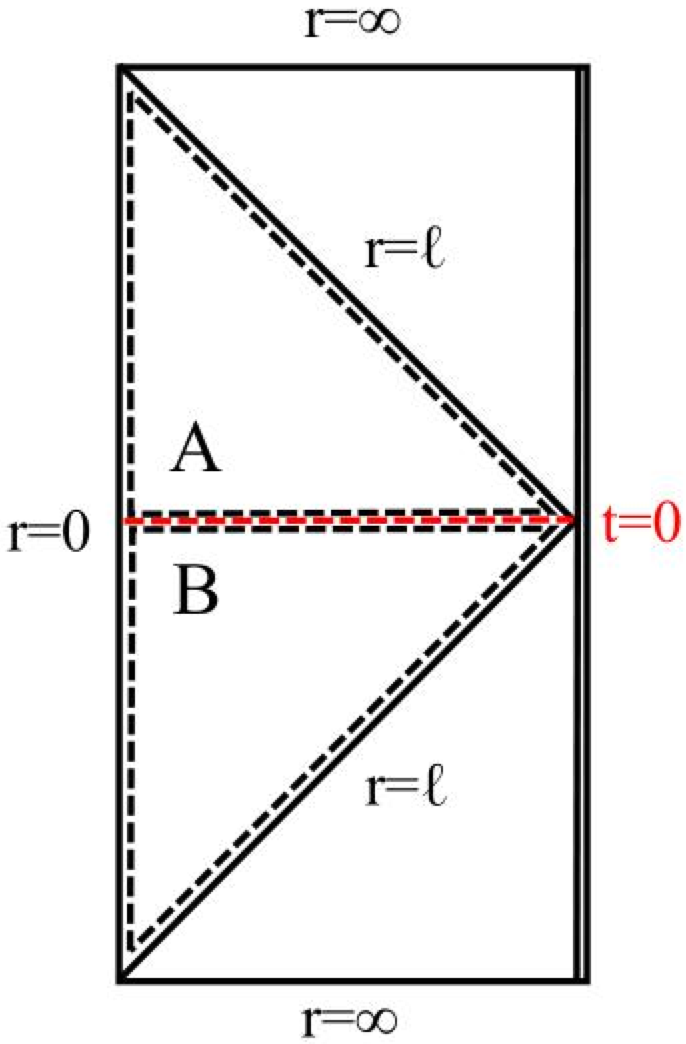
de-Sitter spaces

$$-\left(1-\frac{2M}{r}-\frac{r^2}{\ell^2}\right)dt^2+\frac{dr^2}{1-\frac{2M}{r}-\frac{r^2}{\ell^2}}$$

$$+r^2(d\theta^2+\sin^2\theta\, d\phi^2).$$

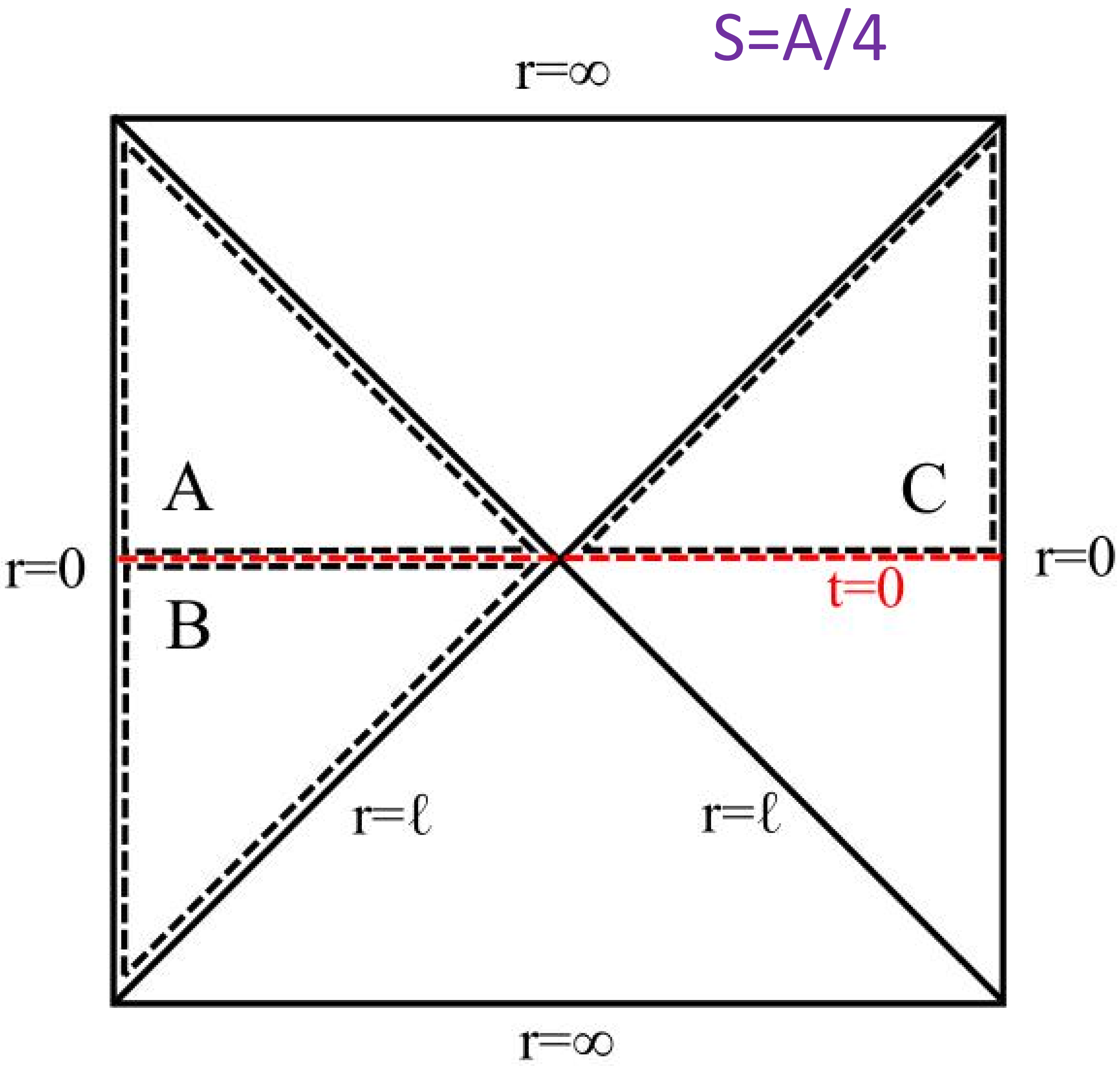


Penrose diagram of $dS[S^3]$

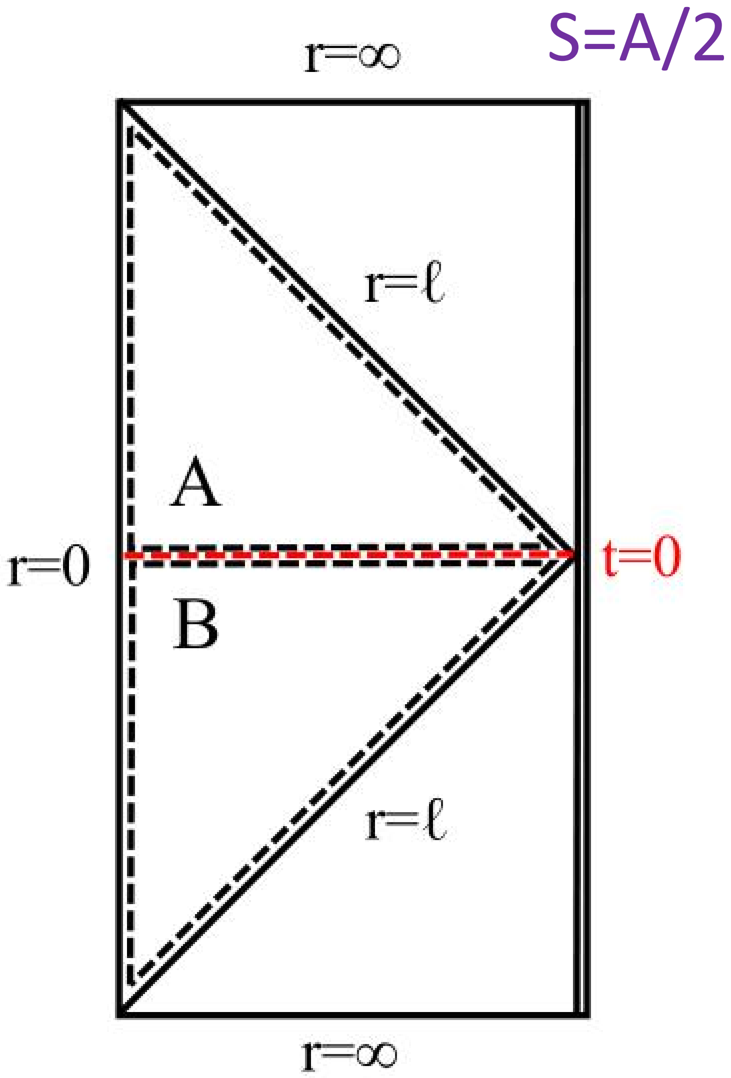


Penrose diagram of $dS[\mathbb{RP}^3]$.

de-Sitter spaces



Penrose diagram of $dS[S^3]$



Penrose diagram of $dS[\mathbb{RP}^3]$.

The Puzzle of Schwarzschild de-Sitter Black Hole

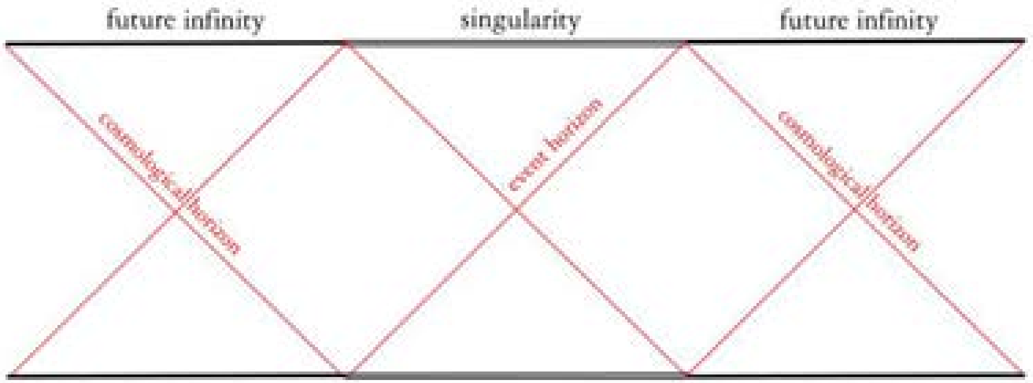


Figure 2. *Left:* the Penrose diagram of a Schwarzschild de-Sitter black hole, SdS[S^3]. Here the diagram extends indefinitely toward the right, as well as the left hand side.

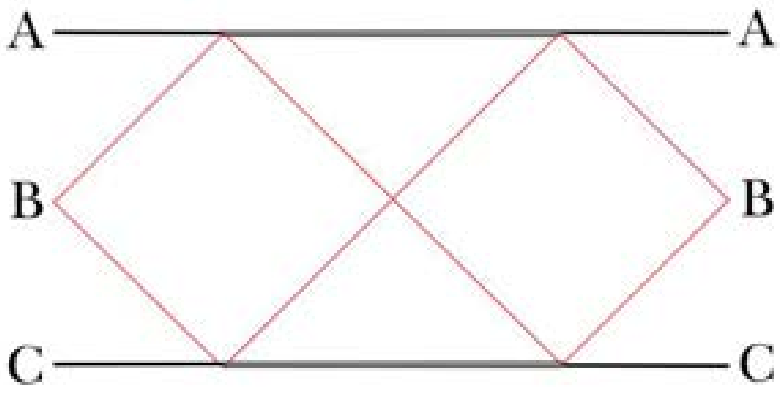


Figure 3. *Left:* the Penrose diagram of a Schwarzschild de-Sitter black hole, SdS[S^3], but with topological identification, so that instead of indefinitely many black holes as in figure 2, we only have one black hole.

No such problem with real projective spatial sections!

Why Do We Care? (1)

In principle, there is observational effect at the *quantum* level: an inertial observer who couples to a free scalar field through a monopole detector could distinguish the difference between these two spaces, although the difference become exponentially small in the distant past or future on the observer's worldline.

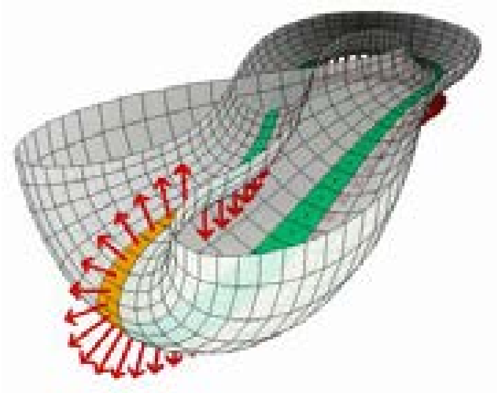
J. Louko and K. Schleich, *The exponential law: Monopole detectors, Bogolubov transformations and the thermal nature of the Euclidean vacuum in* Rp^3 *de Sitter space-time*, Class. Quant. Grav. **16** (1999) 2005 [gr-qc/9812056] [INSPIRE].

P. Langlois, *Causal particle detectors and topology*, Annals Phys. **321** (2006) 2027 [gr-qc/0510049] [INSPIRE].

Why Do We Care? (2)

Alice Universe: An Alice universe is one in which the particle/antiparticle distinction cannot be defined consistently at every point of space (not "charged orientable"). Particle turns into anti-particle after going around the Universe.

Note: Charge is conserved by production of Cheshire Charge: A positron converts into electron, and 2 positive Cheshire charges are created. This then encourages a negative charge, say, antiproton to cross the surface, producing a proton and cancels the Cheshire charges.

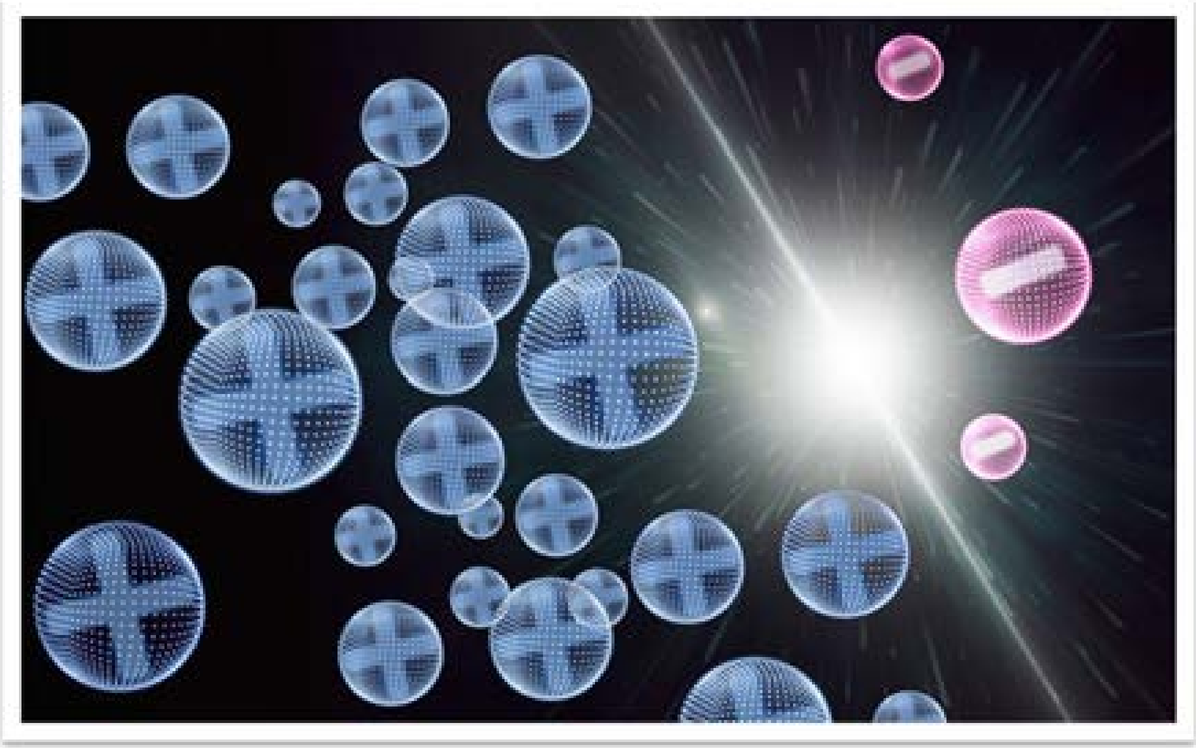


Brett McInnes, Alice Universes, Classical and Quantum Gravity 14 (1997) 2527.

Is Matter-Antimatter Asymmetry caused by a "topological Freeze-Out"?

Whether a universe is Alice, depends on the underlying gauge group and topology.

$\mathbb{R} \times \mathbb{RP}^3$ can be an Alice universe in the context of [SU(2) x SU(2)]U(3) gauge theory.

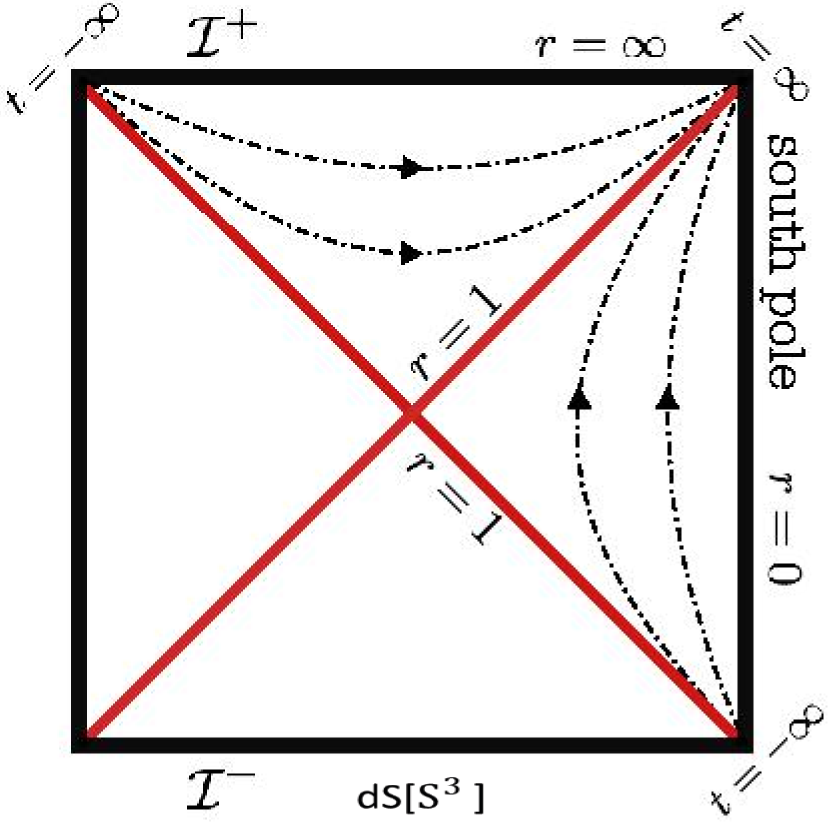


Brett McInnes, Alice Universes, Classical and Quantum Gravity 14 (1997) 2527.

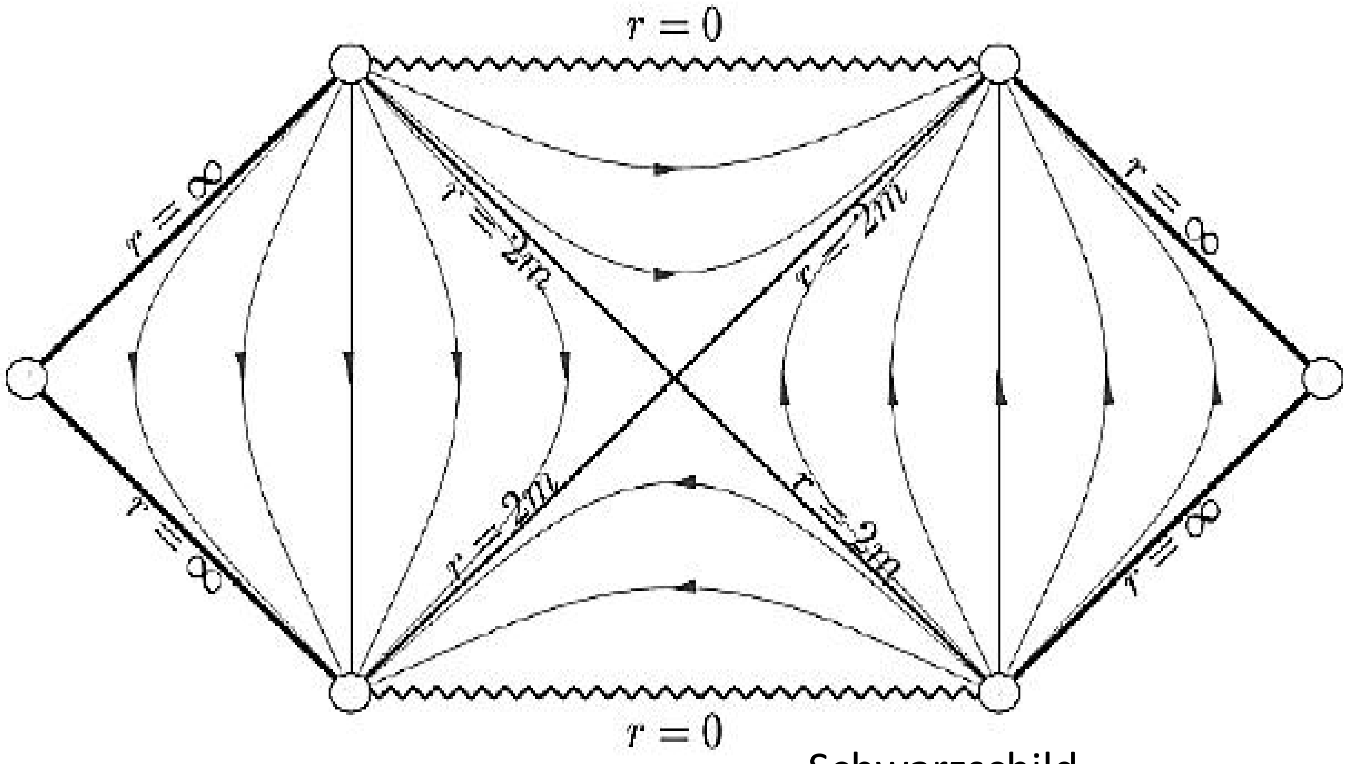
Why Do We Care? (3)

Horizon entropy as entanglement entropy works better with dS[$\mathbb{RP}^3$].

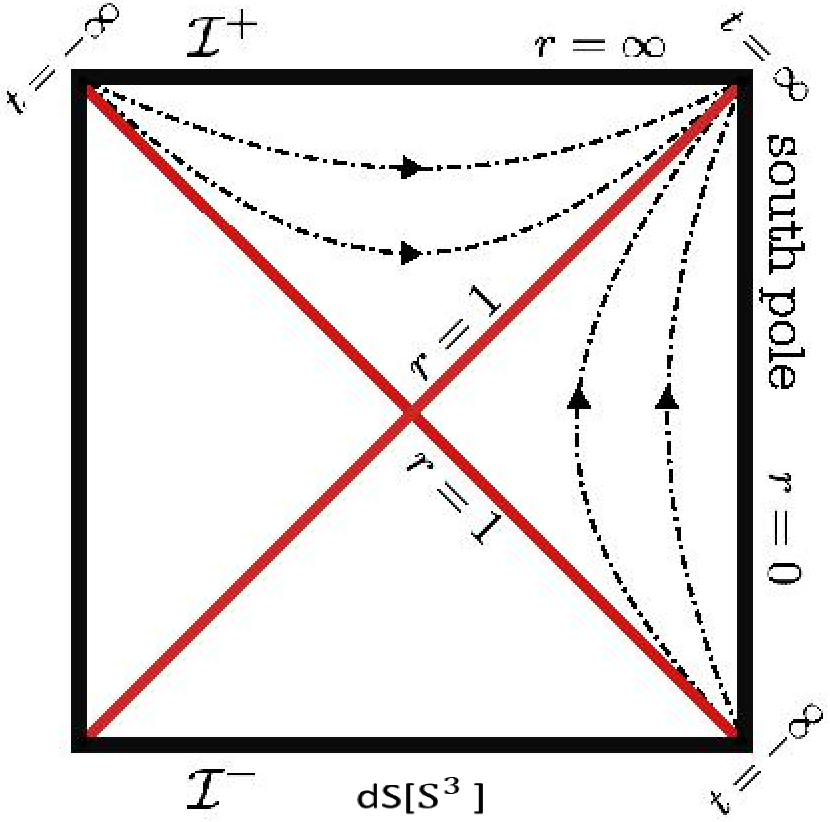
Why interpretation as entanglement entropy is problematic with dS[S^3]:



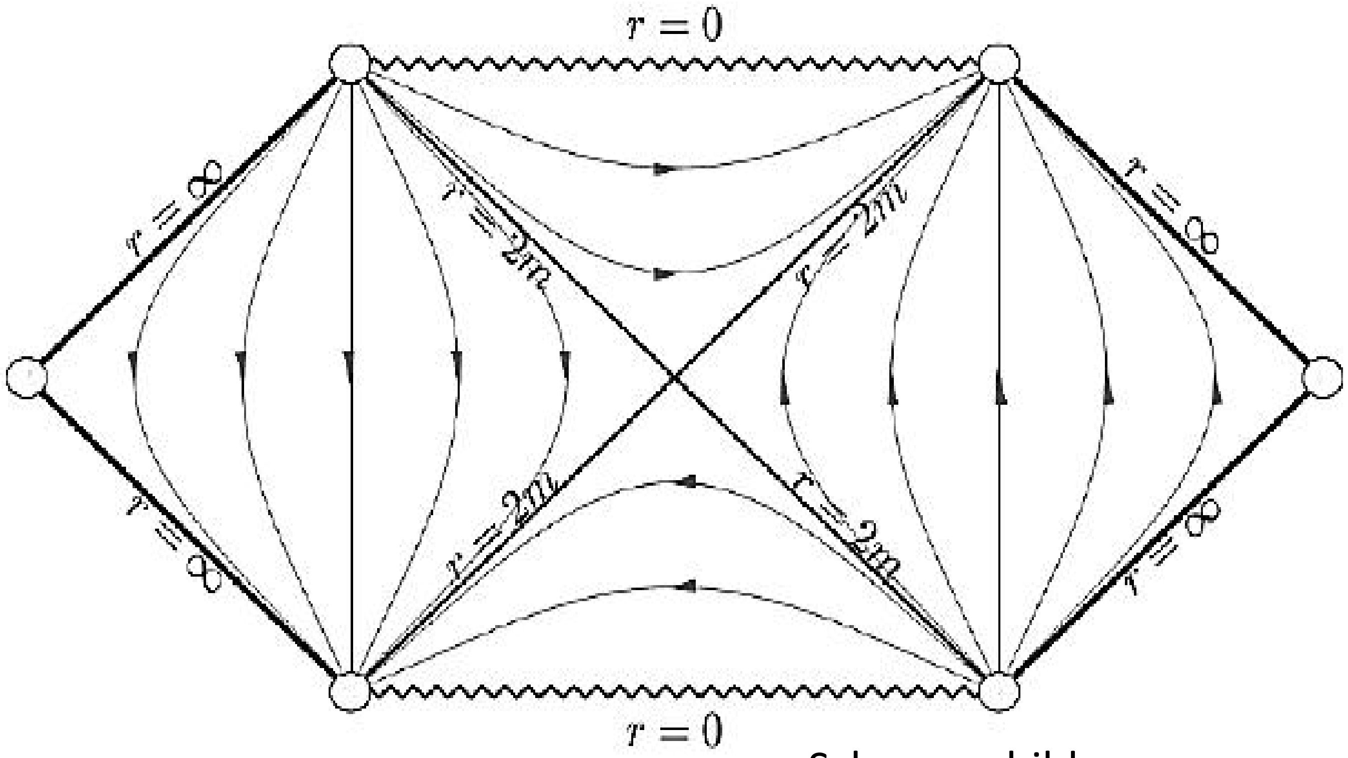
vs



Causally disconnected asymptotic regions can be treated as independent system: Bekenstein-Hawking entropy can be interpreted as the entanglement entropy of the two asymptotically flat regions. (Also, ER=EPR.)



vs



Schwarzschild

Yet to be understood: Subtleties with Wick Rotation

Euclidean version of dS[$\mathbb{R}P^3$] and creation probability: For various technical reasons, EdS[$\mathbb{R}P^3$] is not a manifold but an orbifold.

Journal of Cosmology and Astroparticle Physics
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Instanton tunneling for de Sitter space with real projective spatial sections

Yen Chin Ong[a,b,c,1] and Dong-han Yeom[d]

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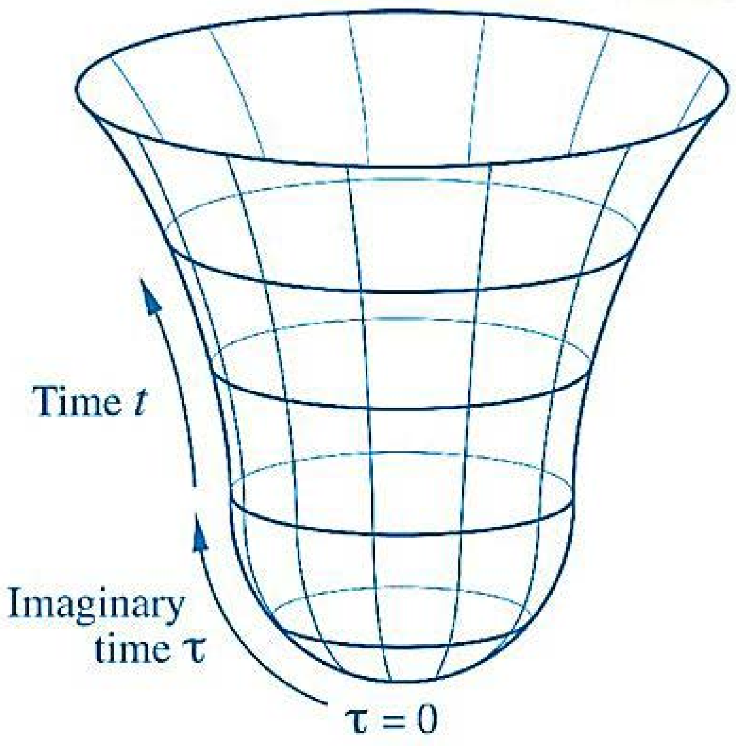

Hawking-Hartle No-Boundary Proposal

We begin not with de Sitter spacetime but rather with the Schwarzschild-de Sitter spacetime discussed above. We take it that the black hole is small ($M << L/(27)^{\frac{1}{2}}$), with energy E = M. There are two horizons, the black hole horizon with radius r_+, and the cosmological horizon with radius r_{++}; we have

$$1 - \frac{2E}{r_{++}} - \frac{r_{++}^2}{L^2} = 0,$$

or

$$E = \frac{1}{2} r_{++} [1 - \frac{r_{++}^2}{L^2}].$$

Thus we may think of r_{++} as varying with E:

$$\frac{dE}{dr_{++}} = -1 \ + \ \frac{3}{2}[1 \ - \ \frac{r_{++}^2}{L^2}]. \tag{11}$$

The area of the horizon in dS($\mathbb{R}P^3$) is of course given by A $= 2\pi r_{++}^2$. Assuming as usual that the entropy is proportional to the horizon area, $S = \zeta A$, we obtain

$$\frac{dS}{dE} = \frac{4\pi\zeta r_{++}}{-1 \ + \ \frac{3}{2}[1 \ - \ \frac{r_{++}^2}{L^2}]}. \tag{12}$$

Now by considering a system consisting of initially well-separated matter and a black hole, Bousso argues that we should take $dE = -dM$. The first law of thermodynamics now gives us

$$T = \frac{1 \ - \ \frac{3}{2}[1 \ - \ \frac{r_{++}^2}{L^2}]}{4\pi\zeta r_{++}} \tag{13}$$

for the temperature of the de Sitter horizon. For a very small black hole, r_{++} is approximately equal to L (the de Sitter horizon radius), and so we have, approximately,

$$T = \frac{1}{4\pi\zeta L}. \tag{14}$$

Now temperature is a strictly local quantity, proportional to the surface gravity [29]. The surface gravities for the cosmological horizons of dS(S^3) and dS($I\!\!RP^3$) are the same — only the areas differ, *not* the radii. If we had done the above calculation in dS(S^3), we would have obtained a temperature of $1/8\pi\zeta L$. Since the two answers must agree, the constant ζ differs in the two cases:

$$\zeta_{I\!\!RP^3} = 2\zeta_{S^3}. \tag{15}$$

Thus for example if the entropy is one quarter of the horizon area in dS(S^3), then it is half the horizon area in dS($I\!\!RP^3$) — which is the same numerical value in each case, namely πL^2. Thus, there is no discrepancy in the value of the entropy in the two cases, only in the way in which that value is computed.